Traces of a quantum antiresonance in a driven system

E. Eisenberg, R. Avigur, and N. Shnerb
Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel
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It has been shown previously that classically chaotic kicked systems, whose unperturbed spectrum possesses one energy scale, exhibit a quantum antiresonance (QAR) behavior. Under the QAR condition, the quantum evolution is completely periodic. In this study we extend the conditions under which this QAR occurs for the case of a two-sided kicked one-dimensional infinite potential well. It is then shown by a perturbative argument that this QAR affects the behavior of the equivalent driven well, namely, the number of periods needed to leave the initial state has a sharp peak around the QAR. We give numerical evidence that the antiresonance persists even for large values of the perturbation parameter. This manifestation of the QAR is experimentally realizable by looking at the absorption spectrum of a quantum well.

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The study of "quantum chaos," i.e., understanding the fingerprints of classical chaos in quantum mechanics [1], has led to the discovery of a variety of new quantum-dynamical phenomena. Several such phenomena occur in time-periodic systems described by the general Hamiltonian

$$H = H_0 + H_1 f(t) , \qquad (1)$$

where H_0 is some time-independent Hamiltonian, H_1 represents a perturbation, and $f(t)$ is periodic with period T, $f(t+T) = f(t)$. In many cases, $f(t)$ is chosen, for simplicity, as a periodic δ function, $f(t) = \Delta_T(t) \equiv \sum_{s=-\infty}^{\infty} \delta(t - sT)$, giving the well-studied class of "kicked" systems. [2–12].

The quantum dynamics of time-periodic systems (1) is governed by their quasienergy (QE) spectrum (i.e., the spectrum of the one-period evolution operator). Different properties of the QE spectrum lead to quantum-dynamical phenomena having, in general, no classical analog. (1) A pure point QE spectrum leads to quantum suppression of chaotic diffusion in the kicked rotator (KR) [2], dynamical localization [5,6], and quasiperiodic recurrences [4]. (2) In cases where there is a mixture of pure point, singular continuous, and absolutely continuous spectra, anomalous quantum diffusion [8,9] occurs. (3) A continuous spectrum manifests itself through quantum diffusion. This happens for the kicked harmonic oscilator ("crystalline" resonance conditions) [12]. (4) Finally, the nongeneric case of quantum resonances [3], i.e., the ballistic (quadratic), rather then diffusive, increase of the energy expectation value with time is due to an absolutely continuous (band) QE spectrum, and occurs in all the kicked systems.

A recently studied phenomenon for time-periodic systems is that of exactly *periodic* recurrences [10,13,14]. This phenomenon is defined, in general, by

$$U^p = e^{-i\beta} , \qquad (2)$$

where U is the one-period evolution operator for (1), $e^{-i\beta}$ is some constant phase factor (a c number), and p is the smallest positive integer for which (2) is satisfied. Thus pT is the recurrence period. This phenomenon, manifesting itself in bounded, periodic variation of expectation values, is diametrically opposite to the quantum resonance (quadratic variation with time). We shall therefore refer to (2) as to the *quantum antiresonance* (QAR) phenomenon.

The simplest class of systems considered are the two-sided kicked rotors (TKR's) [13,14], defined by the Hamiltonian

$$H = \frac{L^2}{2I} + \hat{k} V(\theta) \sum_{s=-\infty}^{\infty} (-1)^s \delta\left(t - \frac{sT}{2}\right) , \qquad (3)$$

where I is the moment of inertia, $\hat{k}$ is the kicking parameter, T is the time period, and $V(\theta)$ is a general periodic and analytic function of the angle θ. Two-sided kicking perturbations such as in (3) were considered in several physical contexts [15] as approximations of sinusoidal driving potentials corresponding to ac electromagnetic fields. By increasing $\hat{k}$ in the classical TKR, one observes the typical transition from bounded to global chaos [13], as in the KR case. The quantum dynamics is governed, as usual, by the evolution operator U in one period, e.g., from $t = -0$ to $t = T - 0$:

$$U = e^{-i\tau \hat{n}^2} e^{ikV(\theta)} e^{-i\tau \hat{n}^2} e^{-ikV(\theta)} , \qquad (4)$$

where $\hat{n} \equiv L/\hbar = -id/d\theta$, $\tau \equiv \hbar T/(4I)$, and $k \equiv \hat{k}/\hbar$.

A most distinctive feature of the quantum TKR is that U becomes the identity operator for $\tau = 2\pi m$, an integer [since the operator $\exp(-i\tau\hat{n}^2)$ in (4) is clearly the identity in this case]. This implies *exactly* periodic recurrences (with period 1) of an arbitrary wave packet [13]. This phenomenon is referred to as quantum antiresonance.

The previously described QAR was based on the fact that the unperturbed evolution between successive kicks is the identity operator so that the opposite sign kicks are canceled. It is therefore clear that the same phenomenon will occur for a two-sided kicked one-dimensional (1D)

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infinite potential well. In the present study, we show the existence of a new antiresonance phenomenon for the case of the linearly kicked one-dimensional potential well. The new QAR occurs when the period of the kicks is *half* the period needed for the more general QAR, as described in Eq. (4). Thus, this QAR is, in some sense, a "period halving" of the former, and specific to this special case, as will be discussed below. It is then shown that this new QAR, namely, the one that occurs when a complete period of the perturbation is compatible with the level spacing frequencies, affects the behavior of the corresponding driven system. In particular, in the region of the QAR condition, the driven system becomes nearly periodic. This may be considered as a "trace" of the exact QAR in the kicked system. Numerical evidence for this effect is presented, and experimental realizations are discussed.

Consider the Hamiltonian

$$H = \frac{P^2}{2m} + \lambda \bar{X} \sum_{m=-\infty}^{\infty} \{\delta(t/T - m) - \delta(t/T - m + 1/2)\}, \tag{5}$$

defined in the infinite well $x \in [0, L]$. We use the dimensionless form, defined by the transformation $\tau = 2\pi t/T$, $X = \bar{X}/L$, and obtain the Schrödinger equation

$$i\frac{d\psi}{d\tau} = \mathcal{H}\psi = \left\{ -\frac{\hbar_{\rm eff}}{2}\partial_x^2 + \frac{\beta}{\hbar_{\rm eff}} X \sum_{m=-\infty}^{\infty} \{\delta(\tau - 2\pi m) - \delta(\tau - 2\pi(m+1/2))\} \right\}\psi, \tag{6}$$

where $\hbar_{\rm eff} = \hbar/(m\omega L^2)$ and $\beta = \lambda/(m\omega^2 L)$. The evolution operator thus takes the form

$$U = \exp(iF) = e^{i\pi \frac{\hbar_{\rm eff}}{2}\partial_x^2} e^{-i\frac{\beta}{\hbar_{\rm eff}}X} e^{i\pi \frac{\hbar_{\rm eff}}{2}\partial_x^2} e^{i\frac{\beta}{\hbar_{\rm eff}}X}. \tag{7}$$

Evidently, since the eigenvalues of the operator $-\partial_x^2$ are $\lambda_n = n^2\pi^2$ with n integer, if $\hbar_{\rm eff}$ takes the values $\hbar_{\rm eff} = 4k/\pi^2$ (k integer), $\exp(i\pi\frac{\hbar_{\rm eff}}{2}\partial_x^2)$ is just the identity operator, and the free evolution of the system between two kicks turns out to be trivial. Since the two kicks are of opposite sign the whole evolution operator is also periodic. This is the usual quantum antiresonance. However, we now show that for this special case, a QAR exists also when $\hbar_{\rm eff} = 2k/\pi^2$. In order to do that, we point out that

$$e^{i\pi \frac{\hbar_{\rm eff}}{2}\partial_x^2} e^{-i\frac{\beta}{\hbar_{\rm eff}}X} = e^{-i\frac{\beta}{\hbar_{\rm eff}}X_I} e^{i\pi \frac{\hbar_{\rm eff}}{2}\partial_x^2}, \tag{8}$$

where $(X_I)_{mn} = X_{mn} e^{-i\pi\frac{\hbar_{\rm eff}}{2}\pi^2(m^2-n^2)}$. Since X_{mn} takes the form

$$X_{mn} = \begin{cases} -\frac{8mn}{\pi^2(m^2-n^2)^2}, & m+n \text{ odd} \\ 0, & m+n \text{ even}, \ m \neq n \\ 1/2, & m = n, \end{cases} \tag{9}$$

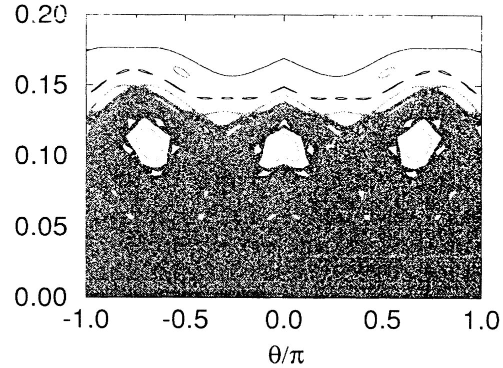

FIG. 1. Stroboscopic map of phase space trajectories for the linearly driven well, for $\beta = 0.01$, and several initial conditions.

it is clear that $(X_I)_{mn} = -X_{mn} + \mathbf{1}$, where $\mathbf{1}$ is the unit matrix, and the whole evolution operator U is then

$$U = e^{-iF} = e^{-i\beta/\hbar_{\rm eff}}, \tag{10}$$

which is the identity operator up to a constant phase. Thus, one obtains a "period halving" of the usual quantum antiresonance.

We now turn to studying the driven version of this problem. This system is classically chaotic [16] as shown in Fig. 1. Its quantum behavior is described by the dimensionless Schrödinger equation

$$i\frac{d\psi}{d\tau} = \mathcal{H}\psi = \left\{ -\frac{\hbar_{\rm eff}}{2}\partial_x^2 + \frac{\beta}{\hbar_{\rm eff}} X \cos(\tau) \right\}\psi. \tag{11}$$

Clearly, due to the continuous character of the perturbation, the argument leading to the exact QAR described above is not valid for this Hamiltonian. However, as we now show, traces of the exact periodicity found in the kicked system in the QAR case are seen in the driven system as well. In fact, kicked systems are widely used as an approximation to cosinusoidally driven perturbations [15]. This can be understood in terms of the relation

$$\sum_{n=-\infty}^{\infty} \left[\delta\left(\frac{t}{T} - n\right) - \delta\left(\frac{t}{T} - \left(n + \frac{1}{2}\right)\right)\right] = 4\sum_{n=1}^{\infty} \cos[(2n-1)\Omega t], \tag{12}$$

through which it is clear that in the limit where one may neglect the effect of the higher frequencies, the kicks are essentially the same as the first cosine term.

It turns out that under the condition $\hbar_{\rm eff} = 2l/\pi^2$ the QAR manifests itself in the perturbative expansion of the time-dependent evolution operator. This can be shown as follows. The transition probability (to order k in β) for a complete period is given in terms of a time-ordered integral of the form

$$A^{(k)}(m \to n) \sim (\beta/\hbar_{\rm eff})^k T \int_0^{2\pi} dt_1 \cdots \int_0^{2\pi} dt_k X_I(t_1) \cdots X_I(t_k) \cos(t_1) \cdots \cos(t_k). \tag{13}$$

Using the matrix representation of $X_I(t)$:

$$(X_I)_{mn}(t) = X_{mn} e^{-it\frac{\hbar_{\rm eff}}{2}\pi^2(m^2-n^2)}, \tag{14}$$

Eq. (13) takes the form

$$A^{(k)}(m \to n) \sim (\beta/\hbar_{\rm eff})^k \sum_{j_1,\cdots j_{k-1}} X_{mj_1} X_{j_1 j_2} \cdots X_{j_{k-1}n} T \int_0^{2\pi} dt_1 \cdots \int_0^{2\pi} dt_k \exp\left(-it_1 \frac{\hbar_{\rm eff}}{2}\pi^2(m^2 - j_1^2)\right)$$
$$\times \exp\left(-it_2 \frac{\hbar_{\rm eff}}{2}\pi^2(j_1^2 - j_2^2)\right) \cdots \exp\left(-it_k \frac{\hbar_{\rm eff}}{2}\pi^2(j_{k-1}^2 - n^2)\right) \cos(t_1) \cdots \cos(t_k). \tag{15}$$

In the antiresonance case, $\hbar_{\rm eff} = 2k/\pi^2$, all the frequencies in the integrals are integers, and thus since the integral is over a complete period the only contribution comes from the zero-mode terms. It is easy to see that even in the most "soft" case, i.e., $l = 1$, the first contribution to the transition amplitudes is at least of order β^3 (if the ground state is populated). This is due to the fact that to order β^k the integrand related to the transition $m \to n$ contains the frequencies $\omega_{mn} \pm 1 \pm 1 \cdots \pm 1$ [k times]. Since the smallest frequency is $\omega_{12} = 3\alpha$, the first- and second-order terms do not have any zero-frequency components. Thus the integral over a whole period vanishes.

In order to confirm the above predictions, we have solved the time-dependent Schrödinger equation numerically, using a quality control Runge-Kutta method, for various values of β and $\hbar_{\rm eff}$ in the antiresonance region. The results are shown in Fig. 2. We plot the inverse of the probability to leave the initial state after one period. This quantity describes approximately the amount of time needed to leave the initial state. It is interesting to note that the QAR traces persist even for large values of β, up to $\beta = 2$. However, as β increases, the position of the antiresonance is slightly shifted.

It is interesting to note that such a model system can in fact be realized experimentally. Modern semiconductor technology has enabled the fabrication of 1D quantum wells [17]. Such a quantum well is fabricated by varying the alloy composition in a compound semiconductor like $Al_xGa_{1-x}As$ along one dimension. Conduction electrons in such structures experience an arbitrarily shaped effective potential in the growth direction while remaining free in the perpendicular plane. Quantum wells are typically $200 - 300$ meV deep with level spacing ΔE between several meV and 150 meV. These systems are of special interest since they can be treated by means of pure quantum mechanical considerations while they are still experimentally accessible. Recently, there has been interest in the behavior of such systems under the influence of an electromagnetic field [16,18–20]. The quantum well structure can be considered as an analogue of an 1D atom, and thus a study of the driven well may help us learn about the interaction of atoms with high-field electromagnetic radiation. In the region where the electric field is strong relative to the level spacing of the well, one obtains a system where nonperturbative effects in light-matter interaction can be studied. In fact, currently there are some experiments carried out at UCSB with the Free Electron Laser (FEL) in which an intense monochromatic far-infrared radiation is applied to a quantum well [21]. Changing the frequency used, the well width and

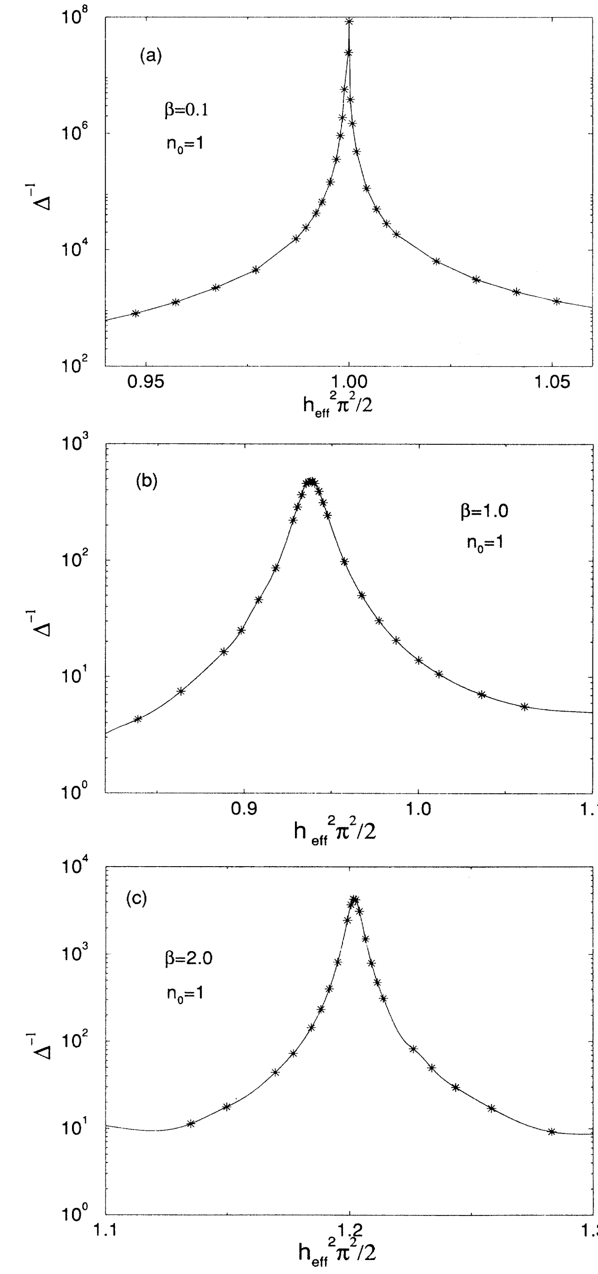

FIG. 2. The inverse of Δ, the probability of escaping from the initial state after one period of the driving force, as a fuction of $\hbar_{\rm eff}$, for various values of β. The stars correspond to the numerical results and the continuous line is an interpolation curve to guide the eye. (a) β=0.1; (b) β=1.0; (c) β=2.0.

the intensity, a wide range of parameters may be accessed, including those that correspond to the QAR. The QAR behavior can therefore be realized experimentally, using a quantum well radiated by a far-infrared laser. One should expect a sharp antipeak in the absorption spectrum of the quantum well.

In conclusion, we have extended the concept of QAR for a case in which the entire period of the perturbation is in resonance with the level spacing frequencies. It has been shown by theoretical arguments and numerical results that this effect persists for driven systems as well. The conditions under which this effect occur can be realized in experiments on quantum wells in far infrared radiation.

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